5.6 Connecting Variation, Slope, and First Differences

You have learned to identify a linear relation from its graph, equation, and table of values. For example, from the graph of a linear relation, you can tell if it is a direct variation or a partial variation and calculate its slope. In addition, you can identify a linear relation from its table of values by calculating first differences.

Consider the distance travelled by a snail over time. Is the graph of this relationship linear? How could you find the slope?

In this section, you will learn how variation, slope, and first differences are connected.

Tools

- grid paper

Investigate

How are variation, slope, and first differences connected?

The table shows the height, compared to the ground, of a snail as it crawls up a pipe.

Time, t (min)	Height, h (m)
0	−3
3	1
6	5
9	9
12	13

1. Graph the relation. Is this a direct variation or a partial variation?

2. Describe the pattern in the t-values. Use first differences to confirm that the relation is linear.

3. Calculate the slope.

4. How does the slope relate to the first differences and the pattern in the t-values?

5. What is the initial value of the height?

6. Write an equation of the line.

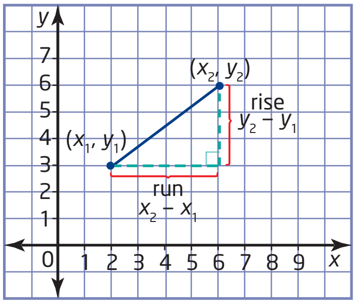
7. **Reflect** Describe how first differences, slope, and partial variation are related.

The slope of a linear relation remains constant. The first differences also remain constant when the changes in the x-values are constant.

The slope, m, of a line can be calculated by dividing the change in y by the change in x.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \end{aligned}$$

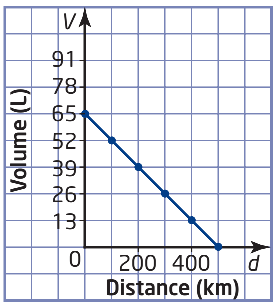
This is sometimes abbreviated as $\frac{\Delta y}{\Delta x}$, which is read as "delta y over delta x." The Greek letter delta is the symbol for *change in*.



The equation of a line has the form $y = mx + b$, where m represents the slope and b represents the vertical intercept, or the value of the dependent variable where the line intersects the vertical axis.

Example 1 Fuel Consumption

The graph shows the relationship between the volume of gasoline remaining in a car's fuel tank and the distance driven.



a) Calculate the slope and describe its meaning.

b) Determine the vertical intercept.

c) Write an equation for this relation.

Solution

a) Use the first two points on the line to calculate the slope.

Use $(x_1, y_1) = (0, 65)$ and $(x_2, y_2) = (100, 52)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{52 - 65}{100 - 0} \\ &= \frac{-13}{100} \\ &= -\frac{13}{100} \end{aligned}$$

Since the slope of a linear relation is constant, I can use any pair of points and the slope will be the same.

The rate of change of the volume of fuel in the tank is $-\frac{13}{100}$ L/km.

The car uses an average of 13 L of gasoline per 100 km driven. This is a negative quantity because the volume of gasoline is decreasing.

b) The vertical intercept is the value of V when $d = 0$. From the graph, $V = 65$ when $d = 0$. Therefore, $b = 65$.

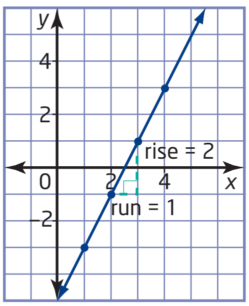
c) This is a partial variation, so its equation has the form $V = md + b$. The equation of this relation is $V = -\frac{13}{100}d + 65$.

Example 2 Slope and the Constant of Variation

Make a table of values and graph the relation $y = 2x - 5$. Draw a right triangle on your graph to find the slope.

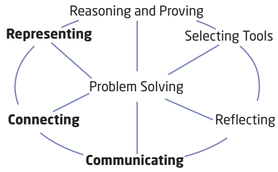
Solution

x	y
1	−3
2	−1
3	1
4	3



$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2}{1} \\ &= 2 \end{aligned}$$

The slope is the same as the constant of variation in the equation $y = 2x - 5$.



The Rule of Four
A relation can be represented in a variety of ways so that it can be looked at from different points of view. A mathematical relation can be described in four ways:
- using words
- using a diagram or a graph
- using numbers
- using an equation

Example 3 Slope and the Equation of a Relation

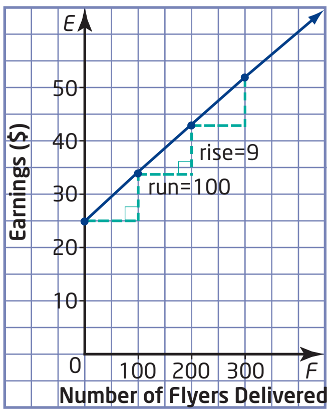
Jacques earns $25 per day plus $9 per 100 flyers for delivering advertising flyers. This is an example of using words to describe a relation. Use the rule of four to describe this relation in three other ways.

Solution

Using numbers: Create a table showing Jacques's earnings for various numbers of flyers.

Number of Flyers, *F*	Earnings, *E* ($)
0	25
100	34
200	43
300	52

Using a graph: Graph this relation.



The graph is a straight line that does not pass through (0, 0). This is a partial variation.

Between any pair of points, there is a rise of 9 for a run of 100. The graph intersects the vertical axis at $E = 25$.

Using an equation:

The relation is linear with $m = \frac{9}{100}$ and an initial value of 25.

This is a partial variation. The equation representing this relation is $E = \frac{9}{100}F + 25$, where E is Jacques's earnings, in dollars, and F is the number of flyers delivered. The slope represents Jacques's rate of pay in relation to the number of flyers delivered.

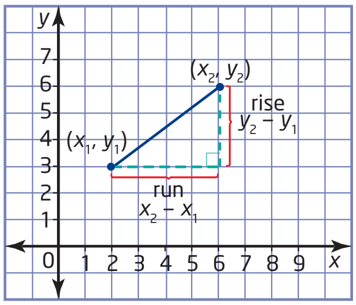
Looking at the table, the F-values change by a constant amount of 100 and the E-values change by a constant amount of 9.

$$m = \frac{\text{change in } E}{\text{change in } F}$$
$$= \frac{9}{100}$$

The initial value of E is 25.

Key Concepts

- Finite differences and the pattern in the x-values can be used to find the slope of a linear relation.
- The constant of variation is also the slope of a linear relation.
- A constant, or average, rate of change can be interpreted as the slope of a relation.
- Slope can be symbolized as $m = \frac{\Delta y}{\Delta x}$, where Δ represents *change in*, or $m = \frac{y_2 - y_1}{x_2 - x_1}$.



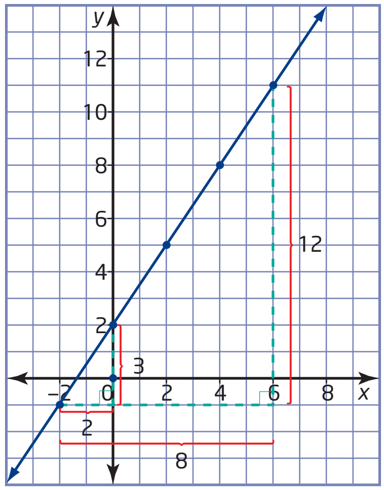
- A line has an equation of the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.
- The Rule of Four can be used to represent a relation in four ways:
  - using words
  - using a diagram or a graph
  - using numbers
  - using an equation

Communicate Your Understanding

C1 The constant of variation and the slope of a relation are the same. Explain why this is true.

C2 Describe the different ways you can find the slope of a linear relation.

C3 How can you find the slope of this line? Explain.



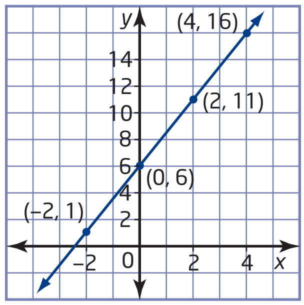
Practise

For help with questions 1 and 2, see Example 1.

1. a) Calculate the slope.

b) Determine the vertical intercept.

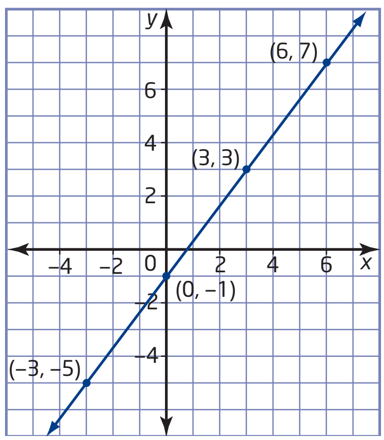
c) Write an equation for the relation.



2. a) Calculate the slope.

b) Determine the vertical intercept.

c) Write an equation for the relation.



For help with question 3, see Example 2.

3. Make a table of values and graph each relation. Draw a right triangle on your graph to find the slope.

a) $y = 2x + 1$

b) $y = -3x + 4$

c) $y = -\frac{3}{2}x$

d) $y = 0.5x + 0.2$

For help with questions 4 to 6, see Example 3.

4. Use the rule of four to represent this relation in three other ways.

a) Use a graph.

b) Use words.

c) Use an equation.

x	y
0	2
1	5
2	8
3	11
4	14

5. Use the rule of four to represent this relation in three other ways.

a) Use a graph.

b) Use words.

c) Use an equation.

x	y
−6	1
−4	6
−2	11
0	16
2	21

6. A house painter charges $400 plus $200 per room to paint the interior of a house. Represent the relation using numbers, a graph, and an equation.

Connect and Apply

7. The cost of a taxi ride is $5.00 plus $0.75 for every 0.5 km.

a) Graph this relation.

b) Identify the slope and the vertical intercept of the line. What do they represent?

c) Is this a direct or a partial variation? Explain.

d) Write an equation relating the cost and the distance travelled.

8. The table shows how the depth of a scuba diver changes with time. Complete the rule of four for the relation by representing it using words, a graph, and an equation.

Time (s)	Depth (m)
0	−50
5	−45
10	−40
15	−35
20	−30

9. y varies directly with x. When $x = 4$, $y = 9$.

a) Find the slope and the vertical intercept of the line.

b) Write an equation for this relation.

c) Graph this relation.

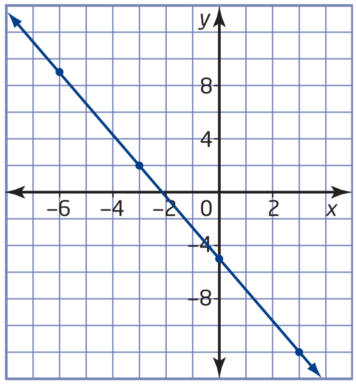
10. y varies partially with x. When $x = 0$, $y = 5$, and when $x = 6$, $y = 8$.

a) Find the slope and the vertical intercept of the line.

b) Write an equation for this relation.

c) Graph this relation.

11. Complete the rule of four for this relation by representing it numerically, in words, and with an equation.



12. Complete the rule of four for the relation $y = 4x - 3$ by representing it numerically, graphically, and in words.

13. A swimming pool is being drained. The table shows the volume of water, in kilolitres, remaining after an elapsed time, in minutes.

Time (min)	0	40	120	180
Volume of Water (kL)	50	40	20	5

a) Confirm that this relation is linear.

b) Graph this relation.

c) Find the slope of the graph as both a fraction and a decimal. Is the slope constant? What does the slope represent?

d) Write an equation for the volume of water in terms of the time.

e) Use your graph or equation to find the volume of water after 60 min.

Achievement Check

14. A company tests the heavy-duty elastic bands it makes by measuring how much they stretch when supporting various masses. This table shows the results of tests on one of the elastic bands.

Mass (kg)	0	2	4	6	8
Length (cm)	6.2	9.6	13.0	16.4	19.8

a) Graph the relation between mass and length.

b) What does the point (0, 6.2) represent?

c) Find the slope of the graph. Is it constant? What does it represent?

d) Write an equation for the length in terms of the mass.

e) Predict how long the elastic band will be when it is supporting a 10-kg mass.

f) If the length for an 8-kg mass were 19.0 cm, how would the answer to part e) change?

Extend

15. This table shows the recommended dosage for a particular drug, based on the patient's mass.

Mass (kg)	Dosage (mg)
40	30
50	35
60	40
70	45
80	50
90	55
100	60
110	65
120	70

a) Write an equation relating the dosage and the mass of the patient.

b) The maximum dosage is 110% of the recommended dosage. Write an equation relating the maximum dosage and the patient's mass.

c) Graph both relations. Compare the graphs.

16. A salesperson's monthly sales and pay for a 4-month period are shown in the table. Determine the salesperson's base salary and percent commission on sales. Describe any assumptions you had to make.

Sales ($)	Salary ($)
15 000	1300
28 000	1560
34 000	1680
17 500	1350